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Research Article

Exploration of pre-service teachers' pedagogical content knowledge in mathematics learning in senior high school based on gender and academic skills

Ma'rufi[1]*, Muhammad Ilyas[2], Salwah[3], Rio Fabrika Pasandaran[4], Muhammad Ikram[5]

Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, Indonesia

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Abstract

In Indonesia, pre-service mathematics teacher education is strictly supervised so that Pedagogical Content Knowledge (PCK) becomes an important aspect to build the quality of learning. This study aimed to explore pre-service mathematics teachers' PCK based on gender and academic skills. To obtain rich and in-depth data, a qualitative approach was used. A total of 70 subjects aged between 19 – 21 years old participated in this study. There were two subjects selected based on their academic skills and gender. Using a grounded theory approach, we conducted a preliminary analysis, open coding, axial coding to obtain the three PCK components, namely Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Student (KS). Research findings revealed that the pre-service teachers' pedagogical content knowledge in terms of knowledge of subject matter was categorized as good in mathematics learning. As for their knowledge of pedagogy, the male subjects presented the concepts by employing the expository strategy, the female subjects with high skills used the guided discovery, and the female subjects with average skill also employed the strategy of expository. In the aspect of knowledge of students, the subjects with average skills overcame students' misconception by explaining the procedures and using the strategy of asking, but the subjects with high academic skills did not only implement the two previous strategies but also used their reasoning behind every procedure of problem-solving that they carried out. These findings can be used as recommendations for the development of mathematics learning.

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Introduction

Many developing countries face challenges in improving the quality of mathematics learning. In 2018, Program for International Students Assessment (PISA) showed that, in terms of mathematical ability, Indonesia ranked 72 out of 78 participating countries. It thus indicates that the Indonesian students' mathematical ability is lower than that of the students in other Southeast Asian countries. Similar cases are also found in Peru (Cueto et al. 2017) where 75% students achieved the lowest scores in mathematical argumentation and decision making for real-life problems. This has resulted in developing countries being very unequal in terms of students' mathematical ability. These results raise a question for researchers in the field of education: what variables of the education system have an impact on improving students' mathematical ability.

[1] Associate Professor, Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia, (marufi@uncp.ac.id), Orcid No: 0000-0002-1379-3287 (Corresponding Author)
[2] Associate Professor, Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia, (muhammadilyas@uncp.ac.id), Orcid No: 0000-0002-0016-6380
[3] Assistant Professor, Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia, (salwah@uncp.ac.id), Orcid No: 0000-0002-9566-3071
[4] Assistant Professor, Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia, (riofp@uncp.ac.id) Orcid No: 0000-0002-1258-4694
[5] Assistant Professor, Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia, (muhammad.ikram@uncp.ac.id), Orcid No: 0000-0002-3763-4299

Recent studies are currently beginning to turn to teacher variables, especially in the pedagogical aspects which play a key role in improving the quality of learning. Regarding this, in this article we present data on one of the aspects described in the literature as Pedagogical Content Knowledge (PCK). PCK is an area of research caused by the lack of the instructional quality in the classroom (Baumert et al. 2010). In developing countries, particularly in Indonesia, PCK has not been explored in depth. Therefore, in this article, we make an attempt to fill that gap by exploring the variables that influence teacher's PCK.

Pedagogical Content Knowledge (PCK) is knowledge included "the way of representing and formulating the subject that makes it comprehensible to others" (Depaepe et al. 2018; Gasteiger et al. 2020; Loewenberg Ball et al. 2008; Ma'Rufi et al. 2019; Ma'rufi et al. 2020; Norton, 2019; Torbeyns et al. 2020). Ball (2008) found categories of PCK consist of knowledge of content and students, knowledge of content and teaching, and knowledge of the curriculum. A teacher is one of the essential components affecting the learning process and learning outcome. A teacher is the most influential factor in students' learning. The forms of teachers' influence are evident when they build motivation and proximity with their students, when they plan systematic learning into a lesson unit, the use of various learning approach, ways to tackle students' difficulties, and even to one that could be directly observed in class are how the teachers present learning so that students could be involved in the learning. These could be generalized into a unique knowledge called Pedagogical Content Knowledge (PCK). According to Kind (Williams & Lockey, 2012), factors affecting the development of PCK include the mastery of learning material, teaching experience, and mastery of emotional attributes such as confidence and support from the work environment.

Hawkins (2012) defines PCK as a kind of particular knowledge happening when teachers integrate the knowledge of mathematical content, teaching, and learning. This knowledge is unique for a specific content taught. Components of PCK in mathematics learning developed by An, Kulm, & Wu (2004) consisted of three components which are: (1) knowledge of content, (2) knowledge of curriculum, and (3) knowledge of teaching. Furthermore, Kilic (2011) said that the four components of Pedagogical Content Knowledge in mathematics learning are Knowledge of Subject Matter, Knowledge of Students, Knowledge of Pedagogy, and Knowledge of Curriculum. Magnusson, Krajcik & Borko (1999) described that PCK is teachers' understanding regarding how they help students to understand a certain topic. It includes knowledge of particular topics, problems, and issues that could be organized and adjusted with students' various skills and interests.

In recent years, teacher's knowledge construction has followed Shulman's model (1986) that covers three domains, including Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK), and Curricular Knowledge. PCK and SMK are perceived as the most determining domains for teacher's knowledge (Carrillo-Yañez et al. 2018). PCK has basically been implemented by teachers as knowledge about effective teaching content. The two main components of this knowledge are the teacher's knowledge of student misconceptions and the teacher's knowledge of learning strategies (Depaepe et al. 2018; Torbeyns et al. 2020). Ball et al. (2008) add that the PCK components are related to Knowledge of Mathematics Learning Standards (KMLS), where teachers need to be aware of curriculum specifications at each level. This allows the teachers to be critical and reflective in considering what students should learn and what material should be taken at a certain level. At the same time, the results of previous research have noted that teacher's knowledge is not only focused on the pedagogical and content aspects of their teaching, but can also be traced to other influencing aspects (for example, beliefs, ability levels, or gender) (Cambell et al. 2014).

Pedagogical Content Knowledge (PCK) analyses are apt to be tailored to the level of students in the class. Gasteiger & Benz (2018) have found that teaching mathematics in early childhood (kindergarten) requires knowledge and skills for teaching mathematics in the classroom because sometimes there is a lack of vocabulary or concepts inherent in learning practices. In addition, PCK can be measured by assessing students' problem solving activities (Csíkos & Szitányi, 2020; Depaepe et al. 2018; Verschaffel et al. 2010). Verschaffel et al. (2010) emphasize that students' mathematics performance is considered to have an influence on learning practices, so it is better to find out how teachers assess students' solutions to mathematical problems. This suggests that research on PCK relies on a cognitive approach to investigate the main components of teacher competence. Torbeyns et al. (2020) formulates from a cognitive perspective that PCK is specific knowledge that is static and can be assessed independently from the classroom context which results in its investigation through a paper-and-pencil test. This perspective, however, has the drawback of limiting our insight into the main characteristics of the true PCK. Depaepe et al. (2013, 2018) have identified two PCK perspectives in the research literature, namely the cognitive perspective which refers to the teacher's knowledge base and the situational perspective which refers to the actions taken in the learning process. These two perspectives complement each other in the field of teaching and teacher development, where the cognitive

perspective refers to tests that measure knowledge independently and the situational perspective supports classroom management to create effective classroom contexts. Various literature reviews suggest that it is necessary to complement the findings that come from a cognitive approach that takes into account certain characteristics of teachers, for example beliefs, ability levels, or gender. Basically, teacher's knowledge, beliefs, ability levels, or gender may be interrelated and influence learning practices.

Gender is one of the unique perspectives in mathematics teaching and learning. The research result of Haciomeroglu & Chicken (2012) explained that in studying mathematics, there were still numerous students (male and female) that view mathematics as a boring subject. Based on the perspective, the aspect of gender in mathematics learning has become the attention among educators. Many opinions said that female students were not quite successful in studying mathematics compared to male students. The female students also almost did not have full interest in theoretical questions like what the male students have. They tend to be interested in practical questions.

The difference in gender does not only affect the difference in mathematical skills but also in how to obtain mathematical knowledge. Dilla, Hidayata, & Rohaeti (2018) explicated the influence of gender in mathematics. They said that it is due to a biological difference in the brain of male and female students revealed through observation. Female, in general, are superior in language and writing, while boys are superior in mathematics due to their better spatial skills. Females, in general, have their attention toward concrete, practical, emotional, and personal things whereas male tend to be interested in intellectual, abstract, and objective things (Geary, Saults, & Liu, 2000).

Geary, Saults, & Liu (2000) described that several researchers believed that the impacts of gender (difference of male and female) in mathematics are caused by a biological difference in the brains of Males and Females. Observation revealed that Females are more superior to boys in language and writing while the males are more excellent in mathematics due to their better spatial skills. Ekawati & Wulandari (2011) identified emotional and intellectual differences between male and female. The former is very aggressive, independent, not emotional, more objective, really like exact sciences, and more logical while the latter is not quite aggressive and independent, more emotional, more subjective, less like the exact sciences, and less logical.

Pre-Service Teachers' Knowledge on Trigonometry

Trigonometry is a branch of mathematics that deals with the sides and angles in triangles. Trigonometry is seen as difficult and confusing because it is related to many disciplines, namely algebra, geometry, and graphics. In the intermediate level curriculum, students have studied trigonometry of algebraic functions. Furthermore, at the university level, they have been introduced to trigonometric functions involving derivatives and anti-derivatives, exponents and logarithms, hyperbolic functions, and sequences. Therefore trigonometry is often used to build new ideas and concepts (Siyepu, 2013, 2015).

Various research studies have revealed that many students are not able to develop their ideas in trigonometry, especially regarding the informal use of algebraic notation. Orhun (2010) further explains that the majority of students in calculus class have low performance in operating trigonometric expressions, for example when operating the multiplication between $\sin x \times \sin x$. Siyepu (2013, 2015) found that students were more likely to generalize the properties of $f(a * b) = f(a) * f(b)$ in all cases, so they frequently make mistakes when writing down $\sin(a + b) = \sin a + \sin b$. Kamber & Takaci (2018) found that students had difficulty understanding the periodic function of trigonometry when faced with inequality problems.

There are two causes for this error. First, the ineffectiveness of instruction in trigonometry learning (Tallman & Frank, 2020). Second, the curriculum does not highlight how students relate the different representations of trigonometric concepts and minimize the use of textbooks that emphasize rote and procedures (Fiallo & Gutiérrez, 2017). The priority is therefore to minimize student errors by designing learning that stimulates students to understand the concept of trigonometry (Mesa & Herbst, 2011). The efforts to minimize problems in teaching trigonometry have resulted in some researchers suggesting that teachers and prospective teachers need to develop quantitative and covariational reasoning (Moore, 2014b, 2014a; Moore et al. 2013), reversible reasoning (Ikram et al. 2020a, 2020b), high order thinking skill (Ilyas et al. 2019; Ma'rufi et al. 2020) in trigonometry learning. Furthermore, Tallman & Frank (2020) add that teachers need to have the disposition ability to support student reasoning by emphasizing the coherence of the basic trigonometric concepts.

Problem of Research

Research on trigonometry mainly focuses on the difficulties and errors made by students in solving trigonometry problems, but rarely discusses teacher's Pedagogical Content Knowledge (PCK) in teaching trigonometry. Other variables may be influenced by gender and academic skills. However, literature that related PCK, gender, and academic skills are rarely studied by other researcher. Therefore, researchers focus on problem of research to explore pre-service mathematics teacher's PCK based on gender and academic skills. Furthermore, main problems of research which formulated throughout this study are:

- How do the PCKs of the pre-service mathematics teacher' differentiations according to their academic skills and gender?

Method

Research Model

The data used in this study were collected from a longitudinal study on pre-service mathematics teachers' Pedagogical Content Knowledge (PCK) in solving trigonometric tasks based on academic skills and gender. To answer the research questions, we collected data related to the activities of the teachers when dealing with trigonometric tasks. Thus, a qualitative approach (Miles, Huberman & Saldana, 2014) was employed in this study. Analysis was focused on two things: the teachers' academic skills and gender. Furthermore, we anticipated other variables that might affect the results which provided recommendations in the end of the study. It is expected that the results of the study can provide students with more opportunities to access quality mathematics learning.

Participants

The subjects consisted of 70 pre-service mathematics teachers, aged between 19 – 21 years old, from level two and three, enrolled in an "internship" program in a university. The steps in selecting the subjects were: identifying all students in the sixth or the third internship (70 students identified), classifying students based on their gender, that is 20 of them were male and the rest of them were female, categorizing them based on their Grade Point Average (GPA) into the category of high and average, and selecting two students from each group based on gender so that the subjects selected were a high and average. The subjects had studied basic and advanced trigonometry. Therefore, we considered that their experience and conceptual knowledge were sufficient. We also assumed that they could provide meaningful data for enriching literature on PCK.

Data Collection Tools

This study used semi-structured individual interviews that lasted for 45 minutes to 1 hour. The interviews followed the suggestions of Miles, Huberman & Saldan (2014) and Goldin (2000) to create an informal and comfortable atmosphere. Most of the interviews were conducted in the classrooms and during lunch breaks. The interview began by discussing the purpose of the interviews and investigating all relevant information about the subject's PCK related to trigonometric tasks. The set time to meet the subject was rather difficult. Therefore, the subjects were interviewed at their convenience.

Each interview was recorded with two cameras, one camera focused on the interaction between the interviewer and the subject, and another focused on the subject's work. The interview guidelines used had been refined in the previous study and were designed in such a way as to become a reference for the interviewer. Furthermore, the subjects were asked to be more explicit about their knowledge of trigonometric material so that the resulting data were more representative. Finally, we transcribed the recorded videos for further analysis using a grounded theory approach.

Throughout the research process, steps were taken to strengthen the trustworthiness of the research findings based on Miles, Huberman & Saldana (2014) framework for establishing trustworthiness. Trustworthiness was enhanced through (a) To ensure in coding the data, each team member individually coded the data for his or her team, the two team members then compared their results, examining any inconsistencies until reaching consensus; and (b) validating the process of coding and recoding of the different categories via discussions with several mathematics education specialists. Data collection was complete when new themes and patterns no longer emerged from the data

Data Analysis

Data collection and data analysis were based on the constructivist perspective. We adopted the view of von Glasersfeld (1995) who mentions that pre-service teachers' PCK consists of a collection of schemes based on previous experience. This perspective implies that as researchers, we did not have direct access to subject knowledge and were only able to model their interpretation of cognitive aspects based on academic skills and gender. Thus, our analysis can construct

a hypothetical model of the subject's PCK on observed behavior, including words, gestures, and the results of the assigned tasks.

Grounded theory (Yin, 2011) was used to analyze the subjects' PCK. The analysis consisted of three stages. The first stage, the initial analysis, began during each interview and after the interview. In each interview, we actively formed initial guesses or assumptions based on the subjects' verbal expressions, gestures, and work results. These assumptions were used to guide the interviewer's follow-up questions. Based on the subject's responses to these follow-up questions, initial assumptions were refined until the data collected were considered sufficient. After each interview, the research team met to equalize perceptions, so that the resulting data were richer in relation to the visible PCK. In particular, after conducting several interviews, the research team found that the subjects had different expressions when expressing their knowledge of the trigonometric material.

Second, after conducting the interviews, we analyzed the subjects' video in detail. Through our preliminary analysis of the subjects' responses via open coding, the interpretations of the subjects converged on our initial expectations. We analyzed the first video by developing code to describe an interesting and relevant aspect of the subjects' PCK. Furthermore, these codes were used to analyze the next video, so that additional, refinement and expansion of the initial code were obtained. This process continued until consistent data were obtained. At the end of this open coding process, three codes stood out, namely: Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Student (KS). These codes characterized the teachers' PCK of trigonometry material.

Third, we used axial coding to refine the terms Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Student (KS). The components of PCK studied were Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Student (KS). The knowledge of subject matter comprised factual knowledge, conceptual knowledge, and procedural knowledge while the knowledge of pedagogy consisted of lesson plan and organization, and the implementation of learning strategy. As for KS, it is knowledge about tackling students' errors and misconceptions. We then compared the characteristics of the subjects' PCK based on academic skills and gender. Through this process, we developed a description of each category of the subjects' PCK that appeared in our analysis. Finally, we re-coded the interview data using refined code and framed the findings on the teachers' PCK.

Results

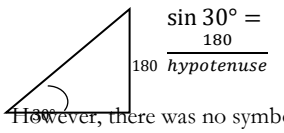
The findings of the study were a deep exploration regarding the components of PCK revealed through mathematics learning on the topic of Trigonometry in senior high school. The topics were following the topic learned by the students during their internship in the school. Every subject carried out their learning two times in senior high school. Their pedagogical content knowledge was described in the following explanation.

PCK of Subjects with High Academic Skills Based on Their Gender

The male and female subjects' pedagogical content knowledge explored in three components were described in the following Table 1 as follows:

Table 1.

PCK of Subjects with High Academic Skills Based on Gender

Components of PCK & Category	Male Subject	Female Subject
Knowledge of Subject Matter		
Conceptual Knowledge	Concepts used by the subject was appropriate such as the ratio formula $\sin \angle A = \frac{opposite\ side\ \angle A}{hypotenuse\ \angle A}$, $\sin 30° = \frac{1}{2}$	The concepts taught by the subject was already appropriate, but there were a concept that was less accurate such as when equating two equations which were $x = c \sin A$ and $x = a \sin C$, she just write them down without prior explanation.
Factual Knowledge	Definition and symbols used were appropriated such as  $\sin 30° = \frac{180}{hypotenuse}$ However, there was no symbol of a right angle on the given triangle so that there was a possibility that the triangle was not a right triangle.	The subject used the definitions and symbols accurately, but there were errors such as when representing the altitudes of the triangle, there was no symbols of the right angle. There was also the writing of symbol like $a = \frac{20}{\sqrt{2}}$ where the equal symbol was not in the middle

Components of PCK & Category	Male Subject	Female Subject
Procedural Knowledge	The procedures of problem-solving explained by the subject were good which start from understanding the problem, representing the problems into the form of figures, employing the formula of the ratio of $\sin \angle A$, and using the value of $\sin 30° = \frac{1}{2}$ to obtain the solution of the problem.	Overall, the subject accurately used the procedures to solve the problem. However, there was still procedures that were not carried out, that was $\frac{a}{\sin A} = \frac{c}{\sin C}$ or $\frac{b}{\sin B} = \frac{c}{\sin C}$
Knowledge of Pedagogy		
Skill in organizing learning	There were two ways regarding how the subject organized the learning which was: (a) dividing students into several groups to facilitate their teamwork, giving task, and building trust with each other, (b) cooperative process supported by students' worksheet to build individual responsibility so that every student was actively involved in learning.	The subject organized learning by 1) arranging the students into small groups (cooperative learning), 2) cooperative learning assisted by worksheet so that students could be more active and there was interaction among students
Skill in implementing a learning strategy	The subject implemented the expository method with the help of powerpoint as instructional media to present the material taught in the class. He presented the material in various ways, both orally and through illustration. The subject also implemented the method of group guidance to solve the exercise. Achieve cooperative learning objectives, he motivated students by asking about the meaning and the importance of teamwork.	Cooperative learning with guided discovery was implemented by the subject to teach the material. However, in the learning process, the teacher still dominated the learning causing the students just receiving the material without constructing the concept on their own. The subject implemented group guidance to solve the exercise.
Knowledge of Students		
Skill in tackling the misconception	The subject skill to tackle misconception was evident when: (1) he employed the strategy of asking to solve students' error; and (2) he used illustration so that the concept presented could be easily understood.	The subject skill to overcome misconception was revealed when: (1) she used questions to tackle students' error; and (2) she explained the relationship among concepts so that the fundamentals of idea development designed by the students could be systematically arranged.

PCK of Subjects with Average Academic Skills Based on Their Gender

PCK of male and female subjects with average academic skills explored in three components of PCK was described in the following Table 2. as follows:

Table 2.

PCK of Subjects with Average Academic Skills Based on Gender

Components of PCK & Category	Male Subject	Female Subject
Knowledge of Subject Matter (KSM)		
Conceptual Knowledge	Concepts of angles relationships presented by the subjects were $\sin(90° - \angle A) = \cos \angle A$ $\cos(90° - \angle A) = \sin \angle A$ $\tan(90° - \angle A) = \cot \angle A$ However, the subject did not emphasize the condition of the concept where it is only applicable in the first quadrant	Concepts taught were appropriate such as $1' = \left(\frac{1}{60}\right)^{\circ}$ 1 degree = 60 minutes $1° = 60' = 3600''$
Factual Knowledge	The subject did not assign a right angle symbol when drawing a right triangle. He wrote down $\tan = \frac{AB}{AC}$ while the correct one should be $\tan C = \frac{AB}{AC}$.	The use of symbols and definition was right like The symbol of degree is ° The symbol of minute is ′ The symbol of second is ″ The symbol of radiant is π

Components of PCK & Category	Male Subject	Female Subject
Procedural Knowledge	Definitions of sine, cosine, and tangent presented were accurate. The subject explained the procedures of problem-solving to students from the symbols writing, explanation about sine values, to the description regarding sides to be compared when we use the law of sine. However, he did not teach the students how to utilize calculators because the angles obtained were not always special angles.	The procedures employed were appropriate and detailed in every steps.
Knowledge of Pedagogy (KP)		
Skill to organize learning	The subject organized the learning by: (1) forming several student groups so that they could cooperate, giving tasks, and building trust with each other; and (2) distributing worksheets that would be completed together to every group.	The subject organized learning by dividing the students into several groups so that students could discuss and solve the exercise given with their peers.
Skill to implement a learning strategy	The subject implemented an expository learning strategy which was orally presenting the material. He also directed the students to do the worksheet in groups	An expository strategy was implemented by the subject to teach the concept. She utilized *powerpoint* as an instructional media so that students could be more focused on the material. She also used the strategy of cooperating to solve the exercise.
Knowledge of Students (KS)		
Knowledge to tackle the misconception	The subject skills to overcome the misconception were revealed when he employed the strategy of asking to explore concepts that the students had not understood yet and when he verbally explained the concepts.	The subject skills in tackling the misconception were evident when: (1) explaining the connection among the concepts so that the fundamental of ideas development planned by the students could be systematically arranged; and (2) employing the strategy of asking.

Discussion and Conclusion

Knowledge of Subject Matter

Knowledge of subject matter is knowledge of the discipline (material knowledge) (Ma'rufi & Ilyas 2017). The knowledge of the material in the study was conceptual knowledge, factual knowledge, and procedural knowledge. The male subjects' conceptual knowledge of the material taught was quite good, but the one with average academic skills paid less attention about the conditions in presenting a concept, for example in the concept of angles relationship $\sin(90° - \angle A) = \cos \angle A$, $\cos (90° - \angle A) = \sin \angle A$ and $\tan (90° - \angle A) = \cot \angle A$, the subject did not emphasize that the concept was applicable in the first quadrant. Female subjects' conceptual knowledge was quite good, the concepts presented were correct.

The factual knowledge that the male subjects possessed was quite good, where the symbols and definitions taught were accurate. Nonetheless, in representing a right triangle, there was no symbol of the right angle leading to the possibility of concept error for students. The use of appropriate and adequate representations greatly contributed to the construction of conceptual understanding.

The female subjects' factual knowledge was categorized as quite good where the symbols and definitions presented were appropriate, but the female subject with high academic skills did not put the symbol that characterizes a right triangle in the process of drawing the triangle. It could lead to an error in the facts presented to the students. Research findings of Ma'rufi et al. (2019)showed that there were 38,4% of junior high school students in Indonesia experiencing factual errors where one of them was interpreting the use of mathematical symbols. As for the procedural knowledge in solving the problem, all subjects' knowledge was categorized as quite good. It was evident in how subjects' explanation to solve the problems was correct.

Knowledge of Pedagogy

Knowledge of pedagogy is pedagogical skill or subjects' skill in teaching. The four subjects organized learning by creating small groups for students (cooperative learning) assisted by students' worksheets. The learning aimed to improve students' teamwork and interaction. Cooperative learning is one that supports effective communication among

learners. To achieve the learning goals, the male subject with high skill motivated students by utilizing video regarding the meaning and the importance of cooperation.

The male subject taught the material by employing expository strategy and the use of various forms of representation such as figures to facilitate learners' understanding. Rangkuti (2014) explicated that: a) representation process involves translation of problems to ideas into a new form; b) representation process also includes the conversion of diagram or physical models into symbols or words; and c) representation process is also used in translation or analysis verbal problems to make the meaning clearer. However, *powerpoint* was utilized by the male subject with high academic skills to assist his teaching. As for the female subject with high skills, she implemented cooperative learning with the method of guided discovery to teach the concepts. However, its implementation was still dominated by the subject so the objectives of the cooperative learning were not achieved. The female subject with average academic skill presented the material by using an expository strategy assisted by *powerpoint* as a learning media. The two subjects tended to employ group guidance as a strategy used to work on the questions in exercises. The aim was to enable students to cooperate in solving the questions given.

Knowledge of Students

Knowledge of student is related to knowledge about students. It involves teachers' knowledge regarding the mathematical concepts that students find it hard to understand, concepts in which students usually have misconceptions, the possibility of the source of students' error, and how to overcome the difficulties and misconceptions. A misconception is caused by students' understanding which is lacking, unstructured, and tentative. Every student had various points of view and was sometimes concluding based on what they had experienced. Several of them concluded things without the correct concepts. The weakness occurred due to their inability to connect or discover the correlation between a concept to other concepts, so they became confused, and a misunderstanding could happen here. The subject tackled the problems with simple but quite systematic ways. There were two ways how the subject overcame the problems, which were through explanations and questions. The strategy of asking the students was also employed by the subject when exploring students' initial knowledge, emphasizing information, and tackling students' misconceptions. These findings were following the research findings of Haryani (2011), describing that the strategy of asking could be viewed as one that forms the mathematical mindset that also trained and habituated the students to do thinking activities including critical thinking.

Through explanation, the subject guided the students to describe the facts and the unknown in the word problem. The subject led the students by asking some short guiding questions. Their skill in asking enabled the students to translate the information in the question and helped them to devise a scheme to solve the problem in a guided manner. This result was following the findings of Killic (2011), suggesting that the teaching, which was based on knowledge of students, did not only deliver procedural information but also helped the students to enhance their conceptual understanding.

In this contribution, we proposed that the Knowledge of Subject Matter, their knowledges are quite the same, but the male subjects with average academic skills lack factual knowledge so there is a possibility of an error in the facts taught to the students. In the Knowledge of Pedagogy, all subjects organize the learning by dividing the students into some groups. There are several strategies used by the subjects to teach the concepts. The male subjects employ the expository strategy with the high academic skills one utilized powerpoint as learning media. The female subject with high academic skills uses the strategy of guided discovery while the other female subject employs the expository strategy assisted by the use of power point. In solving the questions in the exercise, all subjects tend to use the method of group guidance. In Knowledge of Students, all subjects overcome students' misconception problems by asking questions and explaining procedures, but the subjects with high academic skills also encourage students to present the reasoning behind every procedure in their works.

Recommendations

For Further Studies

We see that the three components used in this study have the potential for further investigation regarding teacher Pedagogical Content Knowledge (PCK) on other materials. Meanwhile in this study, we found that there was a significant difference in the pre-service teachers' PCK on each category based on academic skills and gender. There are other indications that may influence and still need to be further explored, namely the curiosity of pre-service teachers regarding the implications of trigonometric material in the real context. Furthermore, each pre-service teacher's line of thinking can be traced to understand the different contexts related to trigonometry. Therefore, further

research can build constructs to better understand how pre-service teachers transfer their ideas to students to understand the concepts in trigonometry.

For Applicants

The findings of this study have implications for teaching and curriculum development, because the interpretation of pre-service teachers regarding trigonometric material is an important aspect of mathematics education at every level. The results of this study can also be used as input for curriculum policy developers, especially that related to trigonometry instructions. It is also necessary to focus on teacher knowledge related to material, pedagogy, and students, because in general, this study shows that the main difficulties faced by teachers are focused on teaching trigonometry. In conclusion, teachers need special skills to construct and build deep knowledge in teaching trigonometry in the classroom.

Limitation of Study

We believe that the limitations of our study point to several aspects. First, the instruments used to collect the data were limited, so future research should add more tasks to ensure that the depth of focus obtained by concentrating on a single task remains. Second, the selection of the subjects was based on academic skills and gender. Therefore, future studies should add other variables, such as cognitive styles that also play a crucial role in determining Pedagogical Content Knowledge (PCK).

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Biodata of the Authors

Associate Professor Dr. Ma'rufi is a lecturer and researcher at the department of mathematics education, facuculty of teacher training and education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia. His research interest is Pedagogical Content Knowledge, Learning Development for Pre-Service Teachers, and Learning Design. **Affiliation:** Universitas Cokroaminoto Palopo, Indonesia **Phone:** +62821-9360-7968 **Email:** marufi@uncp.ac.id **SCOPUS ID:** 57193711083 **WoS ID:** -

Associate Professor Dr. Muhammad Ilyas is a lecturer and researcher at the department of mathematics education, facuculty of teacher training and education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia. His research interest is Metacognitive Ability, Instruction and Learning, and Learning Evaluation. **Affiliation:** Universitas Cokroaminoto Palopo, Indonesia **Phone:** +6282193607965 **Email:** muhammadilyas@uncp.ac.id **SCOPUS ID:** 57193708459 **WoS ID:** -

Assistant Professor Salwah, M.Pd is a lecturer and researcher at the department of mathematics education, facuculty of teacher training and education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia. His research interest is Learning Development. **Affiliation:** Universitas Cokroaminoto Palopo, Indonesia **Phone:** +6285299395428 **Email:** salwah@uncp.ac.id **SCOPUS ID:** 57193709774 **WoS ID:** -

Assistant Professor Rio Fabrika Pasandaran, M.Pd is a lecturer and researcher at the department of mathematics education, facuculty of teacher training and education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia. His research interest is Learning Technology, Algebra, and Geogebra. **Affiliation:** Universitas Cokroaminoto Palopo, **Phone:** +6282347132389 **Email:** riofp@uncp.ac.id **SCOPUS ID:** 57214228653 **WoS ID**: -

Assistant Professor Dr. Muhammad Ikram is a lecturer and researcher at the department of mathematics education, facuculty of teacher training and education, Universitas Cokroaminoto Palopo, South Sulawesi, Indonesia. His research interest is Reversible Reasoning, Calculus Problems, and Thinking Processes. **Affiliation:** Universitas Cokroaminoto Palopo, **Phone:** +6282140018810 **Email:** muhammad.ikram@uncp.ac.id, **SCOPUS ID:** 57216884414 **WoS ID:** -

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